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Prediction of Chamber Pressure Decay Transients during Termination of Solid Propellant Rocket Motors

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Nomenclature

a = reaction constant evaluated from experimental data
A_b = total burning surface area of propellant
A_n = combined area of nozzle throats
A_t = combined area of termination ports
b = diffusion constant evaluated from experimental data
c = constant evaluated from experimental data
C_d = constriction coefficient for orifice-type ports
C_w = mass flow coefficient for choked isentropic flow, i.e.,

$$C_w = \left[\frac{kg}{RT}\left(\frac{2}{k+1}\right)^{(k+1)/(k-1)}\right]^{1/2}$$

M = molecular weight of combustion products
n = constant, evaluated from experimental data
P = chamber pressure at time t
ΔP = 50% reduction from initial chamber pressure, i.e., $\Delta P = \frac{1}{2}P_0$
r = propellant burning rate
R = universal gas constant
t = time after thrust termination
Δt = time interval for 50% reduction from initial chamber pressure
T = stagnation temperature of chamber
V = chamber void volume
w = total mass of gaseous combustion products in chamber
ρ_p = propellant density

Subscript

0 = value at initial termination, i.e., when $t = 0$

Introduction

THE prediction of rocket motor performance during the unsteady-state periods of ignition and termination necessitates an accurate knowledge of the effects of chamber pressure on propellant burning rate. Vieille's law and Summerfield's equation are two such means of relating propellant burning rate to chamber pressure. This note reports an investigation in which Summerfield's equation was found to be more accurate than Vieille's law in predicting the effect of chamber pressure on the burning rate of a composite, double-base, solid propellant rocket motor. This paper also discusses the combination of a mass balance with Summerfield's equation, resulting in an accurate means of predicting pressure decay transients when a solid propellant rocket motor is terminated by opening auxiliary ports in the chamber sides.

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Comparison of Vieille's Law to Summerfield's Equation for a Double-Base Propellant

One often used relationship for predicting the effect of chamber pressure on propellant burning rates is Vieille's law, which states that

$$r = cP^n \tag{1}$$

More recently, Summerfield et al.[1] developed a burning rate equation from diffusional mixing and chemical reaction theory. The equation is intended for use with a heterogeneous solid propellant where the crystals and matrix are assumed to burn at the same rate. Summerfield's equation does not allow prediction of burning rate directly but does predict the pressure dependence of the burning rate to be

$$1/r = (a/P) + (b/P^{1/3}) \tag{2}$$

Summerfield et al. have shown that this equation accurately represents the effect of pressure on the burning rate of selected composite propellant strands for pressures from 14.7 to 1500 psia.

At Hercules Powder Company, a comparison was made between Vieille's law and Summerfield's equation for an aluminized double-base solid propellant for the pressure range of 3 to 1000 psi. It was found that Summerfield's equation more accurately predicted the pressure dependence of the propellant burning rate for this case. Predictions obtained from the two burning rate equations were compared to experimental data accumulated from 65 burning rate experiments with an aluminized double-base solid propellant. These data extended over the pressure range of 3 to 1000 psia and included burning rates from the firing of propellant strands, cylinders, and six different solid propellant rocket motors. Descriptions of some of these motors can be found in Refs. 2 and 3. Vieille's equation correlated with the experimentally determined data for pressures in excess of 200 psi but was unsatisfactory over the entire 3 to 1000 psi range. The burning rates predicted by Summerfield's equation correlated with the experimentally determined burning rates for the full range of chamber pressures. The accuracy of the Summerfield equation is especially notable when considering the wide variety of strand, cylinder, and rocket motor sizes from which the data were obtained. The classified literature[4] contains a detailed presentation and discussion of these data and correlations.

Where rocket chamber pressures vary widely (for example, during ignition and termination), the use of the Summerfield correlation to predict pressure dependence of the burning rate of solid metalized composite or double-base propellants seems to provide the most accurate results. Large errors in prediction of burning rates over wide pressure ranges could

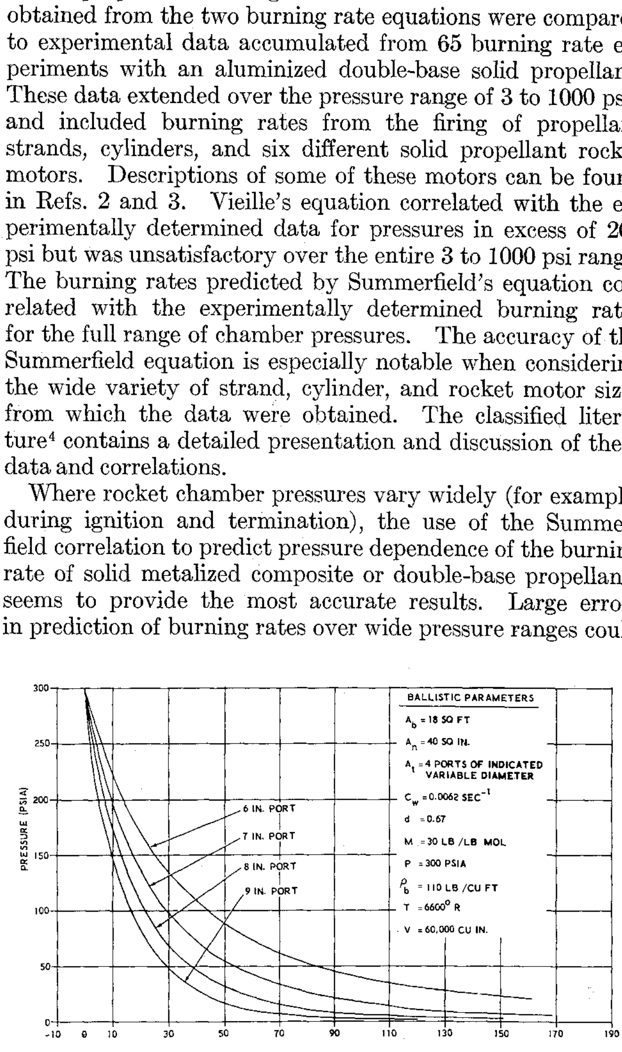

Fig. 1 Predicted effect of port diameter on chamber pressure following thrust termination.

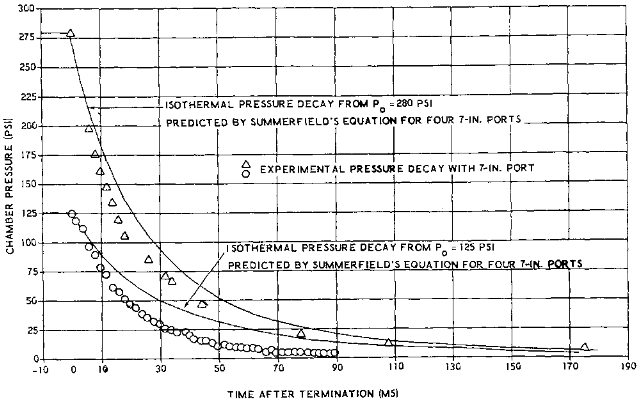

Fig. 2 Comparison of predicted and experimental pressure decay rates.

result when extrapolating more simple empirical burning rate relationships such as Vieille's equation.

Prediction of Pressure Decay Transients during Termination

Kalt[5] studied the rate of chamber pressure decay during termination of a rocket using a solid propellant assumed to obey Vieille's law. The theoretical rocket system under study was assumed to achieve termination by the opening of auxiliary ports in the motor case. This would increase the exhaust area, thus decreasing nozzle thrust by decreasing chamber pressure. Also, proper orientation of the longitudinal axis of the terminator ports relative to the direction of nozzle thrust would provide a reverse thrust. However, as the chamber pressure drops considerably below typical operating pressures, the parameters c and n of Vieille's law are known to change. To overcome this difficulty, Summerfield's equation was combined with a mass balance to derive an equation which describes the pressure decay process during the termination of a double-base, solid propellant rocket motor. At any given time in the pressure decay process, a mass balance indicates that the time rate of change of mass in the chamber[5] is

$$dw/dt = A_b\rho_p r - C_w P(A_t C_d + A_n) \tag{3}$$

By expressing Summerfield's burning rate relationship in the form

$$r = P/(a + bP^{2/3}) \tag{4}$$

and assuming that the perfect gas law applies to the combustion gases, Eq. (3) may be written in terms of pressure as

$$dP = \frac{RT}{MV}\left[A_b\rho_p\left(\frac{P}{a + bP^{2/3}}\right) - C_w P(A_t C_d + A_n)\right] dt \tag{5}$$

After some manipulation, this expression may be integrated to yield

$$t - t_0 = \frac{VM}{RT}\left[\frac{a}{A_b\rho_p - aC_w(A_t C_d + A_n)}\right]\ln\frac{P}{P_0} + \frac{3VM}{2RT}\left\{\left[\frac{a}{A_b\rho_p - aC_w(A_t C_d + A_n)} + \frac{1}{C_w(A_t C_d + A_n)}\right] \times \ln\left[\frac{A_b\rho_p - aC_w(A_t C_d + A_n) - bC_w(A_t C_d + A_n)P_0^{2/3}}{A_b\rho_p - aC_w(A_t C_d + A_n) - bC_w(A_t C_d + A_n)P^{2/3}}\right]\right\} \tag{6}$$

From Eq. (6), it is possible to predict the pressure-time transient during termination of a solid propellant rocket motor. Figure 1 shows this prediction of pressure as a function of time for various port sizes of a solid propellant rocket motor with the indicated ballistic parameters. The time required to eject the port covers was neglected in the calculations.

In Figs. 2 and 3, experimental chamber-pressure transients are compared with those predicted by Eq. (6). The experimental data and predicted chamber pressure decay rates are in close agreement except for one case in Fig. 2 where $P_0 = 125$ psia. This deviation may likely be attributed to propellant extinguishment.

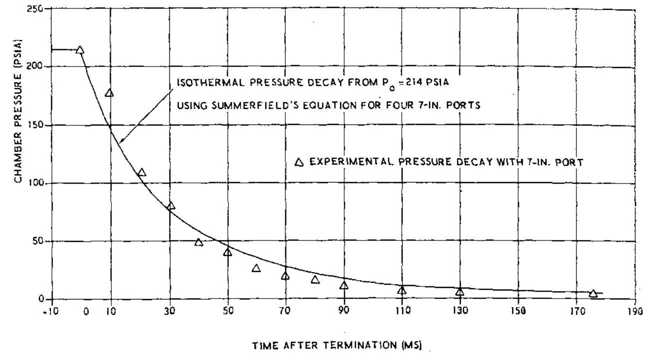

Fig. 3 Comparison of predicted and experimental pressure decay rates.

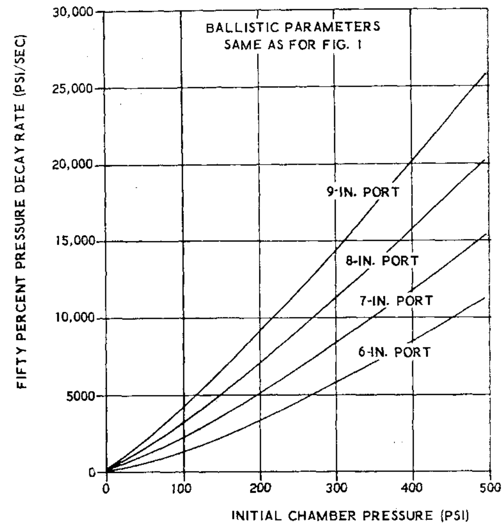

Fig. 4 Predicted effect of initial chamber pressure and port diameter on 50% pressure decay rate.

Figure 4 presents the prediction of 50% pressure decay rate as a function of the initial chamber pressure for different port sizes. This pressure decay rate is defined as $\Delta P/\Delta t$.[6]

The agreement of the predicted pressure decay rates with experimental data indicates that the formula is adequate for predicting the ballistic performance of solid propellant rocket motors during termination. An analogous approach could be applied to the ignition phase of propellant combustion.

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